Constraining crystalline color superconducting quark matter with gravitational-wave data

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We estimate the maximum equatorial ellipticity sustainable by compact stars composed of crystalline color-superconducting quark matter. For the theoretically allowed range of the gap parameter Δ, the maximum ellipticity could be as large as 10^{-2}, which is about 4 orders of magnitude larger than the tightest upper limit obtained by the recent science runs of the LIGO and GEO600 gravitational wave detectors based on the data from 78 radio pulsars. We point out that the current gravitational-wave strain upper limit already has some implications for the gap parameter. In particular, the upper limit for the Crab pulsar implies that Δ is less than $O(20)$ MeV for a range of quark chemical potential accessible in compact stars, assuming that the pulsar has a mass $1.4M_\odot$, radius 10 km, breaking strain 10^{-3}, and that it has the maximum quadrupole deformation it can sustain without fracturing.

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Introduction. When nuclear matter is squeezed to a sufficiently high density, there is a transition from nuclear matter to quark matter. Since the density required for the transition to happen is believed to be not much higher than nuclear-matter density, the dense cores of compact stars are the most likely places where quark matter may occur astrophysically. Except for hot newborn compact stars, it is now generally believed that the deconfined quark matter (if exists) in the interior of compact stars is in a color-superconducting phase [1, 2, 3, 4], in which the quarks form Cooper pairs due to the BCS mechanism (see [5] and references therein for reviews). At sufficiently high densities, the favored pairing phase is the color-flavor-locked (CFL) phase [3], in which pairing between quarks of different colors and flavors is allowed. In the intermediate density regime relevant to the cores of compact stars, it is found that crystalline color-superconducting quark matter is a more favored phase [5, 6, 7, 8, 9, 10]. However, it should be noted that so far the studies on the crystalline phase are based only on phenomenological models of QCD, mainly the Nambu-Jona-Lasinio model [11]. The true ground state of quark matter in the cores of compacts stars is still a matter of debate. Furthermore, there is as yet no study on the construction and stability of a hydrostatic equilibrium stellar model composed of crystalline quark core in general relativity.

While it is still an open question whether quark stars exist in nature, it is interesting to ask how their existence would affect observable phenomena related to compact stars. Furthermore, could we make use of observation data to constrain various theoretical models? In this paper, we show that the observational upper limits on gravitational-wave emission from known pulsars can be used to constrain compact star models composed of crystalline color superconducting quark matter. The prospect of detecting the gravitational-wave signals emitted by quark stars has been considered before (e.g., [12, 13, 14, 15, 16]). However, our work represents the first attempt to make use of real observational data to set constraint on theoretical parameters of the quark matter.

Rotating compact stars are among the most promising gravitational-wave sources for Earth-based interferometric detectors such as LIGO, VIRGO, GEO600, and TAMA300. These objects can emit gravitational waves if they are asymmetric about the rotation axis. A number of mechanisms to give rise to the asymmetry have been proposed, including (1) nonaxisymmetric distortions of the solid crust or core of the star [17, 18, 19, 20]; (2) rotational induced instabilities such as the bar-mode and r-mode instabilities (see [21, 22] for reviews); (3) distortion produced by strong magnetic fields [23, 24, 25]; and (4) free precession of the star [26, 27, 28]. Here we study the nonaxisymmetric distortions of solid compact stars composed of crystalline color-superconducting quark matter and show that the recent observational results by LIGO and GEO600 are already sensitive enough to put constraints on the gap parameter Δ associated with the color-superconducting phase.

Over the past few years, the LIGO and GEO600 detectors have conducted four science runs starting in 2002. In particular, data from the second science run (denoted as S2) have been used to set direct upper limits on the gravitational-wave amplitudes from 28 known pulsars [29]. The lowest gravitational-wave strain upper limit in the S2 run is at the level $\sim 10^{-24}$ and the equatorial ellipticities are $\sim 10^{-5}$. Recently, the LIGO Science Collaboration has presented the latest results for 78 pulsars based on data from the third and fourth science runs (denoted as S3 and S4) [30]. The improved sensitivity of the detectors in the S3/S4 runs leads to the tightest strain upper limit $\sim 10^{-25}$ and equatorial ellipticity $\sim 10^{-6}$.

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For conventional neutron star models, the maximum ellipticity that can be supported by the neutron star's solid crust is about 10^{-7} [18, 19]. The current detectors are still not sensitive enough to probe the upper range of ellipticity permitted by neutron stars. However, the current limits are well into the range permitted by some exotic compact star models. Based on the hypothesis proposed by Xu [31] that quarks might form clusters via a Van der Waals-type interaction, Owen [19] found that solid compact stars composed of such quark-clusters might sustain ellipticity up to $\sim 10^{-4}$. It should be noted that the solid quark-star models considered in this paper, namely, the crystalline color-superconducting quark matter, is more robust than Xu's hypothesis from a theoretical point of view. We find that the maximum ellipticity sustainable by these models could be as large as 5×10^{-2}. This makes the current strain upper limits obtained by the LIGO and GEO600 detectors become more astrophysically relevant and interesting.

Elastic deformations and gravitational-wave emission. A triaxial pulsar, rotating about a principal axis, would radiate gravitational waves at twice the rotation frequency. In the quadrupole approximation, the characteristic strain amplitude is [32]

$$h_0 = \frac{16\pi^2 G}{c^4}\frac{I_{zz} f^2}{r}\epsilon, \tag{1}$$

where f is the pulsar's spin frequency, r is the distance to the pulsar, $\epsilon = (I_{xx} - I_{yy})/I_{zz}$ is the equatorial ellipticity, and I_{ij} is the moment of inertia tensor. The z axis is the rotation axis. Using the definition of the mass multipole moment, $Q_{lm} = \int \rho r^l Y^*_{lm} d^3x$, it can be shown that the ellipticity is related to the $m = 2$ quadrupole moment Q_{22} by

$$\epsilon = \sqrt{\frac{32\pi}{15}}\frac{Q_{22}}{I_{zz}}. \tag{2}$$

In this paper, the pulsar's quadrupole moment is assumed to be due to its elastic deformation in the presence of a solid core. Assuming that the material is everywhere strained to the maximum, the maximum quadrupole moment the star can sustain is given by Ushomirsky *et al.* [18]:

$$Q_{\rm max} = \sqrt{\frac{32\pi}{15}}\sigma_{\rm max}\int \frac{\nu r^3}{g}\left(48 - 14\tilde{U} + \tilde{U}^2 - r\frac{d\tilde{U}}{dr}\right) dr, \tag{3}$$

where $\sigma_{\rm max}$ is the breaking strain, ν is the shear modulus, $g = GM(r)/r^2$ is the local gravitational acceleration, and $\tilde{U} = 2 + d\ln g/d\ln r$. For a uniform-density incompressible model, which is a good approximation to quark stars considered in this paper, $Q_{\rm max}$ is simplified to

$$Q_{\rm max} \approx \frac{13\nu\sigma_{\rm max}R^6}{GM}. \tag{4}$$

It should be noted that in deriving Eq. (3), the self-gravity of the deformation is neglected. However, dropping that approximation would only change $Q_{\rm max}$ by a factor of 2 [20]. This is not large enough to change the implications we shall draw from the estimate of the ellipticity described below.

It is seen from Eq. (3) that more rigid material (i.e., material with a larger shear modulus ν) can lead to a larger quadrupole deformation, which in turn implies a larger ellipticity [see Eq. (2)]. This is the main reason why solid quark stars can sustain much larger ellipticity than conventional neutron stars since the shear modulus of solid quark matter can be a few orders of magnitude larger than that of the neutron star's crust [19] (also see below). In Eq. (3) we define the shear modulus to be half the ratio of the stress to the strain, which has a factor of 2 different from that defined in the original paper by Ushomirsky *et al.* [18]. This definition is consistent with that used by Mannarelli *et al.* [33] in calculating the shear modulus of crystalline color-superconducting quark matter, which is given by

$$\nu = 2.47~{\rm MeV/fm^3}\left(\frac{\Delta}{10~{\rm MeV}}\right)^2\left(\frac{\mu}{400~{\rm MeV}}\right)^2, \tag{5}$$

where Δ is the gap parameter and μ is the quark chemical potential. We remark that this result is obtained by performing a Ginzburg-Landau expansion to order Δ^2. Since the control parameter for the expansion is $[\Delta/(m_s^2/8\mu)] \sim 1/2$ (with m_s being the strange quark mass) for the favored crystalline phase [33], Eq. (5) can only be used to fix the order of magnitude of ν. Higher-order correction terms are suppressed only by about $1/4$.

To describe quark matter within compact stars, the relevant range of μ is 350 MeV $< \mu <$ 500 MeV [33, 34]. In the lower region of this window, Ippolito *et al.* [34] notice that the crystalline phase is more likely to be a two-flavor ($\langle ud\rangle$), rather than the three-flavor ($\langle ud\rangle$ and $\langle us\rangle$) condensate as considered in [33]. In the CFL phase, where the density is high enough so that the effects of the strange quark mass can be neglected, the gap parameter Δ is expected to lie between 10 and 100 MeV [5, 33]. However, for the quark matter to be in the crystalline phase rather than the CFL phase, Mannarelli *et al.* [33] estimate that the reasonable range for Δ is 5 MeV $\lesssim \Delta \lesssim$ 25 MeV. These windows for μ and Δ imply that the shear modulus of crystalline color-superconducting quark matter within compact stars is in the range 0.47 MeV/fm^3 $< \nu <$ 24 MeV/fm^3. For comparison, the shear modulus of the neutron star's crust is of the order 1 keV/fm^3 [18, 33].

The breaking strain $\sigma_{\rm max}$ is highly uncertain. This is the value beyond which solid materials no longer behave elastically. Instead, they would either crack or undergo plastic deformation. For perfect crystals without defects, $\sigma_{\rm max}$ could be as high as 10^{-1} [35]. However, it is also known that $\sigma_{\rm max}$ of real crystals can be several orders of magnitude lower. The discrepancy is explained by the presence of defects (e.g., dislocations) in real crystals. Based on the extrapolation of terrestrial materials, values as high as 10^{-2} has been suggested for neutron star

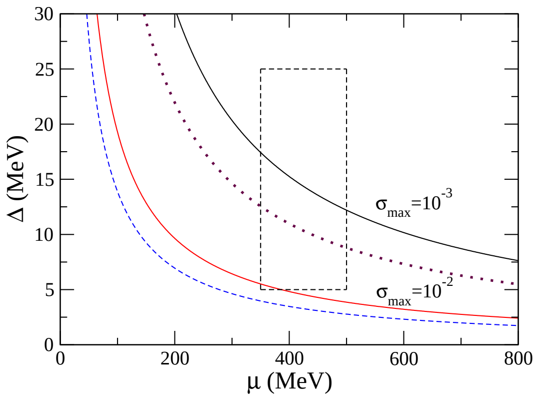

FIG. 1: (color online). The areas above the solid lines are excluded by the direct upper limit for the Crab pulsar obtained from the S3/S4 runs for the cases $\sigma_{\max} = 10^{-3}$ and 10^{-2}. The dotted (dashed) line is the constraint set by the spin-down limit for $\sigma_{\max} = 10^{-3}$ (10^{-2}). The rectangular box is the theoretically allowed region of μ and Δ.

crusts [36, 37], which may also be favored by the enormous energy ($\sim 10^{46}$ erg) liberated in the December 27, 2004 giant flare of SGR 1806-20 according to the magnetar model [38, 39]. In this work, we consider the effects of $\sigma_{\max}$ in the range 10^{-3} to 10^{-2}, values that have been used in the study of compact stars by other authors (e.g., [18, 19, 27, 38]).

Using the shear modulus given in Eq. (5), the maximum quadrupole moment for a solid quark star in the crystalline color-superconducting phase can be estimated by Eq. (4). However, the moment of inertia I_{zz} is needed to calculate the corresponding maximum equatorial ellipticity. Using the empirical formula for strange stars given by Bejger and Haensel (see Eq. (10) of [40]), Eq. (2) gives the maximum equatorial ellipticity as

$$\epsilon_{\max} = 2.6\times 10^{-4}\left(\frac{\nu}{\mathrm{MeV/fm^3}}\right)\left(\frac{\sigma_{\max}}{10^{-3}}\right)\left(\frac{1.4M_\odot}{M}\right)^2 \times\left(\frac{R}{10\ \mathrm{km}}\right)^4\left[1+0.14\left(\frac{M}{1.4M_\odot}\right)\left(\frac{10\ \mathrm{km}}{R}\right)\right]^{-1}. \quad (6)$$

We use the values $M = 1.4M_\odot$, $R = 10$ km, and $\sigma_{\max} = 10^{-2}$ in order to compare with [19], in which Owen obtained $\epsilon_{\max} \sim 2\times 10^{-4}$ for solid strange stars (with the quarks clustered in groups of about 18). For the estimated range of the shear modulus given above, we find that $\epsilon_{\max}$ could be as large as $\sim 5\times 10^{-2}$ for solid quark stars in a crystalline color-superconducting phase. This relatively large value of $\epsilon_{\max}$ is about 4 orders of magnitude larger than the tightest upper limit obtained by the combined S3/S4 result for the pulsar PSR J2124-3358 [30].

Constraints set by the Crab pulsar. Eq. (1) suggests that the observational upper limits on h_0 obtained from known isolated pulsars can be used to set a limit on ϵ assuming a value of I_{zz}. However, the moment of inertia is very sensitive to the poorly known dense matter equation of state (EOS). It can change by a factor of 7 depending on the stiffness of the EOS [40]. Alternatively, with Eq. (2), one can use Eq. (1) to set a limit on the pulsar's quadrupole moment without assuming a value of I_{zz} [41]. The limit can in turn set a constraint on the shear modulus of crystalline color-superconducting quark matter by Eq. (4). In particular, with the expression (5) for ν, we can define an exclusion region in the $\Delta-\mu$ plane by the the following constraint:

$$\Delta\mu \lesssim 7.3\times 10^4\ \mathrm{MeV}^2\left(\frac{10\ \mathrm{km}}{R}\right)^3\left(\frac{1\ \mathrm{Hz}}{f}\right) \times\left[\left(\frac{\tilde{h}_0}{10^{-24}}\right)\left(\frac{M}{1.4M_\odot}\right)\left(\frac{10^{-3}}{\sigma_{\max}}\right)\left(\frac{r}{1\ \mathrm{kpc}}\right)\right]^{1/2}, \quad (7)$$

where $\tilde{h}_0$ is the observational upper limit on h_0 for a given pulsar.

Under the assumption that the pulsar is an isolated rigid body and that the observed spin-down of the pulsar is due to the loss of rotational kinetic energy as gravitational radiation, one can also obtain the so-called spin-down limit on the gravitational-wave amplitude $h_{\mathrm{sd}} = (5GI_{zz}|\dot{f}|/2c^3r^2f)^{1/2}$, where $\dot{f}$ is the time derivative of the pulsar's spin frequency [30]. As it is expected that the strain amplitude satisfies $h_0 \lesssim h_{\mathrm{sd}}$ in general, we can derive a constraint on the product $\Delta\mu$ based on the spin-down limit:

$$(\Delta\mu)_{\mathrm{sd}} \lesssim 2.1\times 10^5\ \mathrm{MeV}^2\left(\frac{10^{-3}}{\sigma_{\max}}\right)^{1/2}\left(\frac{M}{1.4M_\odot}\right)^{3/4} \times\left(\frac{10\ \mathrm{km}}{R}\right)^{5/2}\left(\frac{|\dot{f}|}{10^{-10}\ \mathrm{Hz\ s^{-1}}}\right)^{1/4}\left(\frac{f}{1\ \mathrm{Hz}}\right)^{-5/4} \times\left[1+0.14\left(\frac{M}{1.4M_\odot}\right)\left(\frac{10\ \mathrm{km}}{R}\right)\right]^{1/4}, \quad (8)$$

where we have used Eq. (10) of [40] for the moment of inertia for strange stars.

In [30] it is reported that the gravitational-wave strain upper limit for the Crab pulsar ($\tilde{h}_0 = 3.1\times 10^{-24}$) is the closest to the spin-down limit (at a ratio of 2.2). For the other pulsars, the direct observational upper limits are typically at least 100 times larger than the spin-down limits. For pulsars in globular clusters, in which cases the spin-down measurement is obscured by the cluster's dynamics, the gravitational-wave observations provide the only direct upper limits. In the following, we shall focus on the constraints set by the observational data for the Crab pulsar.

The S3/S4 results for the Crab pulsar are plotted in Fig. 1. In the figure, the solid lines represent Eq. (7) when the equality holds for the breaking strain $\sigma_{\max} = 10^{-3}$ and 10^{-2}, assuming that $M = 1.4M_\odot$ and $R = 10$ km. The pulsar's spin frequency is $f = 29.8$ Hz and the distance is $r = 2$ kpc. For comparison, the dotted and dashed lines are the constraints obtained by the spin-down limit [Eq. (8)], respectively, for $\sigma_{\max} = 10^{-3}$ and

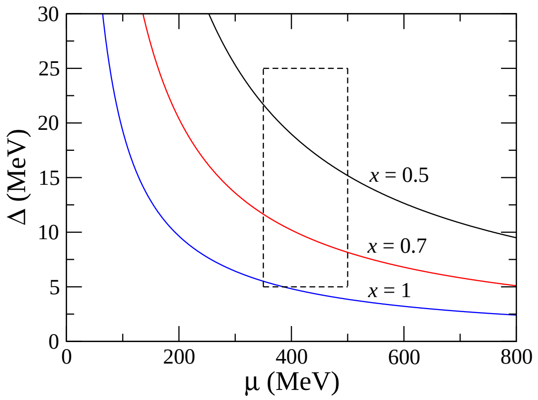

FIG. 2: (color online). The dependence of the exclusion region on the size of the crystalline core. The mass, radius, and breaking strain are fixed at $M = 1.4M_\odot$, $R = 10$ km, and $\sigma_{\rm max} = 10^{-2}$. The parameter $x = R_{\rm c}/R$, where $R_{\rm c}$ is the radius of the core.

10^{-2}, with the pulsar's spin-down rate $|\dot{f}| = 3.73 \times 10^{-10}$ Hz s^{-1}. In the figure, the region to the right of each curve is excluded by the corresponding constraint. The rectangular box (in dashed line) in Fig. 1 encloses the theoretically allowed ranges of μ and Δ in compact stars as discussed above. It is seen that the direct observational upper limits (solid lines) can already place a strong constraint on the possibility that the Crab pulsar might be a solid star composed of crystalline color superconducting quark matter. For the fiducial values $M = 1.4M_\odot$, $R = 10$ km, and $\sigma_{\rm max} = 10^{-3}$, the gap parameter Δ is restricted to be less than $\sim O(20)$ MeV for the range of quark chemical potential relevant to compact stars. Furthermore, the more extreme estimate $\sigma_{\rm max} = 10^{-2}$ excludes essentially the entire theoretically allowed range of Δ.

So far we have assumed that the whole star is composed of crystalline quark matter. However, a more realistic compact star model would be a crystalline quark-matter core surrounded by a nuclear-matter fluid layer, which is then followed a thin crust. As a first step to investigate the effects of a smaller crystalline core on the constraint (7), we assume that the quadrupole deformation (and hence the gravitational wave emission) is due only to the crystalline core, without considering the respond of the outer fluid layer and thin crust. The maximum quadrupole moment (4) is replaced by $Q_{\rm max} \approx 13\nu\sigma_{\rm max}R_{\rm c}^6/GM_{\rm c}$, where $M_{\rm c}$ and $R_{\rm c}$ are, respectively, the mass and radius of the crystalline core. To relate $Q_{\rm max}$ to the global parameters M and R of the star, it is necessary to know the mass distribution of the outer fluid layer, which depends on the nuclear-matter EOS. For simplicity, we assume that the density falls off as $\rho(r) = \rho_{\rm c}[1-(r/R)^2]/[1-(R_{\rm c}/R)^2]$ in the outer fluid layer (i.e., for $R_{\rm c} \le r \le R$), where $\rho_{\rm c}$ is the density of the core [42]. The quadrupole moment can then be expressed as $Q_{\rm max} \approx 13\nu\sigma_{\rm max}R^6\alpha(x)/GM$, where $x = R_{\rm c}/R$, $\alpha(x) = x^6[1-(1-x^2)^{-1}(1-3x^2/5-2x^{-3}/5)]$, and $\alpha(1) \equiv 1$. Accordingly, the constraint (7) is modified by multiplying a factor $\alpha(x)^{-1/2}$ on the right hand side of the equation. The results for three different values of x are plotted in Fig. 2, with $M = 1.4M_\odot$, $R = 10$ km, and $\sigma_{\rm max} = 10^{-2}$ being fixed. The figure shows that a smaller crystalline core would make the extreme estimate $\sigma_{\rm max} = 10^{-2}$ more compatible with the current upper limit for the Crab pulsar. However, the effect due to the respond of the outer fluid layer to the internal deformation deserves further investigation.

Summary and discussion. In this paper, we point out that the S3/S4 runs of the LIGO and GEO 600 gravitational-wave detectors are already sensitive enough to put an interesting constraint on solid quark-star models composed of crystalline color superconducting quark matter. We have estimated that the maximum equatorial ellipticity sustainable by these stellar models could be as large as $\sim 10^{-2}$. This is about 2 orders of magnitude larger than that obtained by Owen [19] for solid quark star models composed of quark-clusters, without taking into account the effects of color superconductivity. We have used the direct observational gravitational-wave strain upper limit for the Crab pulsar in the S3/S4 runs to constrain the poorly known gap parameter Δ in the crystalline color-superconducting phase. For $\sigma_{\rm max} = 10^{-3}$, we conclude that the Crab pulsar could be made entirely of crystalline quark matter if Δ is less than $O(20)$ MeV. For the extreme estimate $\sigma_{\rm max} = 10^{-2}$, the Crab pulsar is unlikely to be a complete solid quark star. However, it could still contain a smaller crystalline quark core in this case. The direct observational upper limit for the Crab pulsar should beat the spin-down limit in the fifth science run of the LIGO detectors [30]. This promises to put a stronger constraint on the theoretical models considered in this paper.

Finally, we conclude with a few remarks. (1) It should be noted that the existence of crystalline quark matter inside compact stars is still a matter of debate. As mentioned before, this phase of quark matter has only been studied in phenomenological models of QCD and no analysis on the gravitational stability of such stellar configuration has been carried out. Our work suggests a way to put constraint on such exotic compact star models based on gravitational-wave observation. We also note that Anglani *et al.* [43] have recently studied the cooling rate of these compact stars. This can provide another observational constraint. (2) A general argument against the existence of quark stars is that (fluid) quark-star models are inconsistent to the behavior of pulsar glitches [44]. The standard explanation for glitches in the conventional neutron star model involves the pinning and unpinning of large numbers of superfluid vortices to the solid crust [45, 46, 47]. But the significant development of the effective theory of QCD over the past few years suggests that the quark matter inside compact stars is likely to be simultaneously superfluids and rigid solids, which are the two crucial conditions for the standard model of glitches (see [6, 33] for discussion and [48] for the study of the

superfluid mode in the crystalline quark-matter phase). (3) One uncertainty in our estimate is the value of the breaking strain $\sigma_{\rm max}$. One should keep in mind that the values $\sigma_{\rm max} = 10^{-3} - 10^{-2}$ used in this paper probably lie in the upper end of the theoretical range [36, 37]. We also assume that the stellar material is everywhere strained to the maximum in the analysis. As pointed out by Owen [19], since a given pulsar may not be strained to the maximum, thus no upper limit on $\epsilon_{\rm max}$ can ever rule out any theoretical model. But the credibility of the model will face serious challenge as the number of observed pulsars increases and tighter limits on $\epsilon_{\rm max}$ are placed.

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